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An analytical and numerical study for thermosolutal nanofluid convection using revised model☆

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Summary The conservation equations for binary nanofluid system are solved to get eigenvalue equation. Valid approximations are used to simplify the complex expressions for analytical results. Oscillatory motions are not possible and hence mode of convection is invariably through stationary mode. Numerical computations are carried out for water based nanofluids to analyze solutal effects on the stability of the system using the software Mathematica. Higher conductivity and density of copper makes it more stable than alumina.

Introduction

Abnormal thermal conductivity enhancement in nanofluids relative to the base fluid motivated Buongiorno (2006) to develop convective transport model for nanofluids. Using this model, various studies on the stability of a horizontal nanofluid layer were reported (Tzou, 2008; Yadav et al., 2012; Gupta et al., 2015) by assuming nanoparticle volume fraction to be constant along the boundaries. It was shown that the stability of a nanofluid layer can be increased or decreased depending on bottom or top heavy distribution of nanoparticles. Recently the problem is revisited (Nield and Kuznetsov, 2014; Agarwal, 2014) by taking zero nanoparticle flux along the boundaries which is a more realistic approach and it was found that the presence of nanoparticles destabilizes the layer under this assumption. We have studied the influence of solute on nanofluid convection using revised model (Nield and Kuznetsov, 2014) which is a triple diffusion problem and leads to complex expression for oscillatory motions. Valid approximations are used to simplify the lengthy expressions to derive analytic results. Further, numerical computations are made to analyze the effects of solute Rayleigh number, Dufour parameter and Soret

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parameter on metallic (copper) and non-metallic (alumina) nanoparticles in water based nanofluid.

Problem formulation

Let us consider a horizontal binary nanofluid layer of thickness d. The temperature and solute concentration at the lower and upper boundaries are T_1 and $T_0(T_1 > T_0)$, C_1 and $C_0(C_1 > C_0)$, respectively and nanoparticle flux is zero along the boundaries of the layer. The relevant equations for the system Gupta et al. (2015) and Nield and Kuznetsov (2014) are

$$\nabla \cdot \boldsymbol{u} = 0, \tag{1}$$

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}\right) = \mu\nabla^2\boldsymbol{u} - \nabla p + [\phi\rho_p + (1-\phi)\{\rho(1-\beta_C(C-C_0) - \beta_t(T-T_0))\}]\boldsymbol{g}, \tag{2}$$

$$\rho c\left[\frac{\partial T}{\partial t} + \boldsymbol{u}\cdot\nabla T\right] = \rho_p c_p\left[D_B\nabla T\cdot\nabla\phi + \frac{D_T}{T_0}\nabla T\cdot\nabla T\right] + k\nabla^2 T + \rho c D_{TC}\nabla^2 C, \tag{3}$$

$$\frac{\partial \phi}{\partial t} + \boldsymbol{u}\cdot\nabla\phi = D_B\nabla^2\phi + \frac{D_T}{T_0}\nabla^2 T, \tag{4}$$

$$\frac{\partial C}{\partial t} + \boldsymbol{u}\cdot\nabla C = D_S\nabla^2 C + D_{CT}\nabla^2 T. \tag{5}$$

In the above equations various parameters are: velocity $\boldsymbol{u} = (u_1, u_2, u_3)$, time t, nanoparticle volume fraction ϕ, coefficient due to Brownian motion D_B, coefficient due to thermophoresis D_T, temperature T, pressure p, viscosity μ, volumetric coefficient due to solute β_C, volumetric coefficient due to heat β_T, gravitational acceleration $\boldsymbol{g}$, density ρ, specific heat c, conductivity k, nanoparticle density ρ_p, nanoparticle specific heat c_p, Dufour type diffusivity D_{TC}, solute concentration C, solute diffusivity D_S and Soret type diffusivity D_{CT}. Let

$$(x, y, z) = \frac{(x, y, z)}{d}, \quad t = \frac{t\alpha_f}{d^2}, \quad \phi = \frac{\phi}{\phi_b}, \quad \boldsymbol{u} = \frac{\boldsymbol{u}d}{\alpha_f}, \quad p = \frac{pd^2}{\mu\alpha_f}, \quad T = \frac{(T-T_0)}{(T_1-T_0)}, \quad C = \frac{(C-C_0)}{(C_1-C_0)}, \tag{6}$$

where $\alpha_f = k/\rho C$ and ϕ_b is the reference value of nanoparticle volume fraction. Using Eqs. (1)–(6) become

$$\nabla \cdot \boldsymbol{u} = 0, \tag{7}$$

$$\frac{1}{Pr}\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}\right) = -\nabla p + \nabla^2\boldsymbol{u} - Rm\hat{k} + R_A T\hat{k} - Rn\phi\hat{k} + \frac{Rs}{L_s}C\hat{k}, \tag{8}$$

$$\frac{\partial T}{\partial t} + \boldsymbol{u}\cdot\nabla T = \frac{N_B}{L_e}\nabla\phi\cdot\nabla T + \frac{N_A N_B}{L_e}\nabla T\cdot\nabla T + N_{TC}\nabla^2 C + \nabla^2 T, \tag{9}$$

$$\frac{\partial C}{\partial t} + \boldsymbol{u}\cdot\nabla C = \frac{1}{L_s}\nabla^2 C + N_{CT}\nabla^2 T, \tag{10}$$

$$\frac{\partial \phi}{\partial t} + \boldsymbol{u}\cdot\nabla\phi = \frac{1}{L_e}\nabla^2\phi + \frac{N_A}{L_e}\nabla^2 T, \tag{11}$$

where various non-dimensional numbers are: Prandtl number $Pr = \mu/\rho\alpha_f$; fluid Lewis number $L_e = \alpha_f/D_B$; solute Lewis number $L_s = \alpha_f/D_S$; Rayleigh number $R_A = \rho\boldsymbol{g}\beta_T d^3(T_1 - T_0)/\mu\alpha_f$; solute Rayleigh number $Rs = \rho\boldsymbol{g}\beta_C d^3(C_1 - C_0)/\mu D_s$; nanoparticle Rayleigh number $Rn = (\rho_p - \rho)\phi_b\boldsymbol{g}d^3/\mu\alpha_f$; basic-density Rayleigh number, $Rm = \rho\boldsymbol{g}d^3/\mu\alpha_f$; diffusivity ratio $N_A = D_T(T_1 - T_0)/D_B T_0\phi_b$; density increment, $N_B = (\rho C)_P\phi_b/\rho C$; Dufour parameter $N_{TC} = D_{TC}(C_1 - C_0)/\alpha_f(T_1 - T_0)$; Soret parameter

$$N_{CT} = \frac{D_{CT}(T_1 - T_0)}{\alpha_f(C_1 - C_0)}. \tag{12}$$

Method of analysis and discussion of results

In the initial state, fluid's velocity is zero; temperature, nanoparticle volume fraction and concentration of solute are varying along z-axis only. We get initial solution of Eqs. (7)–(11) as

$$\boldsymbol{u} = 0, \quad \frac{d\phi}{dz} = N_A z, \quad T = C = 1 - z. \tag{13}$$

Let us impose small perturbations $(\tilde{\boldsymbol{u}}, \tilde{p}, \tilde{T}, \tilde{C}, \tilde{\phi})$ on (13) and use normal mode analysis as

$$(\tilde{u}, \tilde{T}, \tilde{C}, \tilde{\phi}) = (\tilde{u}_3, \tilde{T}, \tilde{C}, \tilde{\phi})(z)\exp(ik_x x + ik_y y + st), \tag{14}$$

where s is a growth rate. Further, using one term residual method for free-free boundaries, we get eigenvalue equation. For non-oscillatory motions put $s = 0$ in eigenvalue equation, this gives the expression for R_A as

$$R_A = \frac{J^3(1 - N_{CT}N_{TC}L_s) - a^2 Rs(1 - N_{CT})}{a^2(1 - L_s N_{TC})} - RnN_A(1 + L_e). \tag{15}$$

where $a = (k_x^2 + k_y^2)^{1/2}$ and $J = \pi^2 + a^2$. Clearly, the presence of nanoparticles destabilizes the system significantly and do not affect the critical wave number. For oscillatory motions, we put $s = i\omega$ in the eigenvalue equation. Let us assume that Lewis number and Prandtl number are very large and Dufour and Soret parameters as negligible, we get ω^2 as negative. Thus oscillatory motions are not possible. Table 1 shows the values of nanofluid parameters for copper and

Table 1 Physical properties and different parameters for nanoparticles under consideration.

Nanoparticles	ρ (kg/m^3)	k (W/m K)	Rn	N_A
Cu (copper)	9000	401	0.008	0.5
Al_2O_3 (alumina)	3970	40	0.003	5

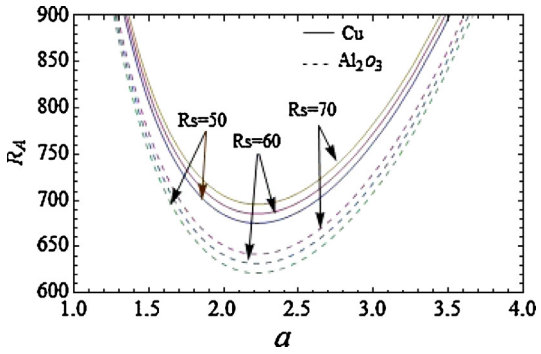

Figure 1 Impact of solute Rayleigh number on Rayleigh number.

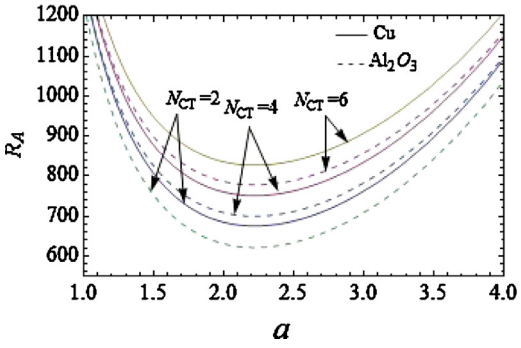

Figure 2 Impact of Soret parameter on Rayleigh number.

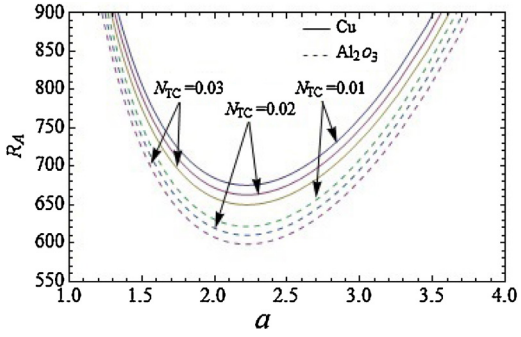

Figure 3 Impact of Dufour parameter on Rayleigh number.

alumina nanoparticles to study the impact of various solute parameters on the stability of the system. Figs. 1 and 2 show the stabilizing effect of solute Rayleigh number and Soret parameter, respectively while destabilizing influence of Dufour parameter is depicted in Fig. 3. It is noteworthy that copper nanoparticles are more stable than alumina in water based nanofluid.

Conclusions

Thermosolutal nanofluid convection is studied analytically and numerically using normal mode technique with the assumption that nanoparticle flux is zero along boundaries of the layer. Oscillatory motions are not possible and mode of convection is stationary. Presence of nanoparticles destabilizes the system significantly. Copper nanoparticles in water based nanofluids are more stable than alumina. The solute Rayleigh number and Soret parameter stabilizes the system while Dufour parameter destabilizes it.

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